ESTIMATION OF THE HEAT TRANSFER COEFFICIENT IN THE SPRAY COOLING OF CONTINUOUSLY CAST SLABS

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ABSTRACT

An inverse problem involving the estimation of the heat transfer coefficient at the surface of a plate, with no information regarding the functional form of the unknown, is solved by applying the conjugate gradient method with adjoint equation. This paper is part of an experimental and numerical simulation of the actual spray cooling process in continuous casting machines. Results obtained with simulated measurements, for the estimation of the time and spatial variation of the unknown spray heat transfer coefficient are summarized. The conjugate gradient method is found to provide accurate estimates for the unknown, even for functions containing sharp corners and discontinuities, which are the most difficult to be recovered by inverse analysis. The effects of number and location of sensors on the inverse problem solution are also addressed on the paper.

NOMENCLATURE

A	dimensionless length of the plate
B	dimensionless width of the plate
Bi	dimensionless heat transfer coefficient
$C_p(\theta)$	dimensionless temperature-dependent specific heat
d	direction of descent given by equation (14.b)
e_{RMS}	RMS error defined by equation (23)
J	functional defined by Eq. (4)
J'	gradient of the functional given by Eq. (13)
$K(\theta)$	dimensionless temperature-dependent thermal conductivity
S	number of sensors
X, Y, Z	dimensionless coordinates

GREEKS

β	search step size given by Eq. (15)
$\Delta\theta$	sensitivity function; solution of the sensitivity problem given by Eqs. (7)
γ	conjugation coefficient given by Eq. (14.c)
λ	Lagrange multiplier; solution of the adjoint problem given by Eqs. (10)
μ	dimensionless measured temperature
θ	dimensionless temperature; solution of the direct problem given by Eqs. (1)
σ	standard deviation of the measurements
τ	dimensionless time
τ_f	dimensionless final time

SUBSCRIPTS

s	refers to the sensor number
o	reference value
ε	perturbed quantity

SUPERESCRIPTS

*	dimensional property
k	number of iterations

INTRODUCTION

The spray cooling techniques utilized in continuous casting have direct influence on the temperature distribution in the slabs, as well as on the local thickness of the solidifying shells. Thermal stresses arise in continuously cast slabs due to irregular surface cooling, while mechanical stresses come into picture because of the pressure exerted by the machine rolls. Hence, the ability to accurately control and predict the behavior of such a cooling system can reduce defects caused by thermal and mechanical stresses in continuously cast products (Hibbins and Brimacombe, 1984, Kohno et al, 1984).

Different studies can be found in the literature on the estimation of the heat transfer coefficient of spray cooling systems, and how it is affected by different parameters, including average water flux, slab surface temperature, distance of the spray to the slab, etc. (Brimacombe et al, 1984, Hibbins and Brimacombe, 1984, Mizikar, 1970). However, such studies were based on average values for the heat transfer coefficient, or the analysis did not involve the solution of an inverse problem, but a comparison of measured and estimated temperatures by trial-and-error.

In this paper we solve the inverse problem of estimating the heat transfer coefficient of an air-mist spray, by using a *function estimation approach* based on the *conjugate gradient method with adjoint equation*. The heat transfer coefficient is allowed to vary with the position over the surface of the slab, as well as with time. The conjugate gradient method of function estimation is a powerful iterative technique, which has been successfully applied to the solution of linear and non-linear inverse problems (Jarny et al, 1991, Orlande and Ozisik, 1993, 1994, Alifanov, 1994, Huang et al, 1995, Dantas and Orlande, 1996, Machado and Orlande, 1997). We use here simulated experimental data in order to assess the accuracy of such a method, as applied to the estimation of the heat transfer coefficient. Different functional forms were tested in order to generate the simulated data, including those containing sharp corners and discontinuities, which are the most difficult to be recovered by inverse analysis. The use of simulated data also permitted the planning of an experimental apparatus, currently under construction, with respect to the number and location of sensors required to obtain accurate estimates for the unknown function.

PHYSICAL PROBLEM

The physical problem considered here involves a laboratory simulation of the actual cooling process of continuously cast slabs. A steel plate is heated up to temperatures of the order of 1000 ^{o}C. Its top surface is then cooled by an air-mist spray, and transient temperature recordings are taken at several locations inside the plate. The plate lateral surfaces are kept insulated, while the transient temperature of the bottom surface is measured with an infrared radiation pyrometer.

The estimation of the heat transfer coefficient of the air-mist spray, by using the conjugate gradient method with adjoint equation, consists of the following basic steps: direct problem, inverse problem, sensitivity problem, adjoint problem, gradient equation, conjugate gradient method of minimization, stopping criterion and computational algorithm.

We present below the details of the each of these distinct steps.

DIRECT PROBLEM

The direct problem is concerned with the determination of the temperature field in the plate, when the spray heat transfer coefficient, as well as the physical properties, initial condition and

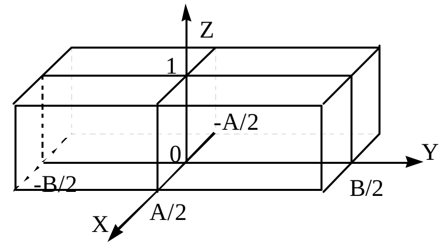

Figure 1 - Geometry and Dimensionless Coordinates

other parameters appearing in boundary conditions are known. The mathematical formulation of this heat conduction problem is given in dimensionless form by

$$C_p \frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial X}\left(K \frac{\partial \theta}{\partial X}\right) + \frac{\partial}{\partial Y}\left(K \frac{\partial \theta}{\partial Y}\right) + \frac{\partial}{\partial Z}\left(K \frac{\partial \theta}{\partial Z}\right) \text{ in } \quad (1.a)$$

$-A/2<X<A/2$, $-B/2<Y<B/2$, $0<Z<1$, for $\tau>0$

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{at } X=-A/2,\ X=A/2, \text{ for } \tau>0 \quad (1.b,c)$$

$$\frac{\partial \theta}{\partial Y} = 0 \quad \text{at } Y=-B/2,\ Y=B/2, \text{ for } \tau>0 \quad (1.d,e)$$

$$\theta = \varphi(X, Y, \tau) \quad \text{at } Z=0, \text{ for } \tau>0 \quad (1.f)$$

$$K \frac{\partial \theta}{\partial Z} + Bi\theta = 0 \quad \text{at } Z=1, \text{ for } \tau>0 \quad (1.g)$$

$$\theta = f(X, Y, Z) \quad \text{for } \tau=0, \text{ in } -A/2<X<A/2, \quad (1.h)$$

$-B/2<Y<B/2$, $0<Z<1$

Figure 1 illustrates the geometry and coordinates, while various dimensionless groups are defined as

$$A = \frac{a}{c}, \quad B = \frac{b}{c}, \quad X = \frac{x}{c}, \quad Y = \frac{y}{c}, \quad Z = \frac{z}{c} \quad (2.a\text{-}e)$$

$$\tau = \frac{K_o^* t}{\rho^* C_o^* c^2}, \quad Bi = \frac{hc}{K_o^*}, \quad \theta = \frac{T - T_\infty}{T_o - T_\infty} \quad (2.f\text{-}h)$$

where a, b and c are the length, width and thickness of the plate, respectively, while T_∞ is the cooling fluid temperature and T_o is a reference temperature for the plate. In order to write the direct problem in dimensionless form, we assumed the temperature dependence of thermal conductivity and specific heat to be in the form:

$$K^*(T) = K_o^* K(\theta) \quad (3.a)$$

$$C_p^*(T) = C_o^* C_p(\theta) \quad (3.b)$$

where K_0^* and C_0^* are reference values for thermal conductivity and specific heat, respectively, while $K(\theta)$ and $C_p(\theta)$ are dimensionless functions of θ. The slab density ρ^* was assumed constant. The superscript "*" above refers to dimensional physical properties.

INVERSE PROBLEM

For the inverse problem, the dimensionless heat transfer coefficient of the air-mist spray, $Bi(X,Y,\tau)$, is regarded as unknown. Such a function is to be estimated by using the transient readings of S temperature sensors located inside the plate, at positions (X_S, Y_S, Z_S), s=1,…,S, during the time interval $0 \le \tau \le \tau_f$. An estimate for $Bi(X, Y, \tau)$ is obtained so that the following functional is minimized:

$$J\left[Bi\left(X,Y,\tau\right)\right] = \frac{1}{2}\int_{\tau=0}^{\tau_f} \sum_{s=1}^{S} \left[\theta\left(X_S, Y_S, Z_S, \tau; Bi\right) - \mu_S(\tau)\right]^2 d\tau \quad (4)$$

where $\theta(X_S, Y_s, Z_S, \tau; Bi)$ and $\mu_s(\tau)$ are the estimated and measured temperatures at the measurement locations, respectively. The estimated temperatures are obtained from the solution of the direct problem by using an estimate for $Bi(X, Y, \tau)$.

In order to apply the conjugate gradient method for minimizing the functional given by Eq. (4), we need to develop and solve two auxiliary problems, known as the *sensitivity* and *adjoint problems*, as described next.

SENSITIVITY PROBLEM

The sensitivity problem is developed by assuming that the temperature $\theta(X, Y, Z, \tau)$ is perturbed by an amount $\varepsilon\Delta\theta(X, Y, Z, \tau)$, when the Biot number $Bi(X, Y, \tau)$ is perturbed by $\varepsilon\Delta Bi(X, Y, \tau)$, where ε is a real number. Due to the non-linear character of the problem, a perturbation on temperature causes perturbations on the temperature dependent properties, as well. Thus, we can write the following perturbed quantities:

$$Bi_\varepsilon(X,Y,\tau) = Bi(X,Y,\tau) + \varepsilon\Delta Bi(X,Y,\tau) \quad (5.a)$$

$$\theta_\varepsilon(X,Y,Z,\tau) = \theta(X,Y,Z,\tau) + \varepsilon\Delta\theta(X,Y,Z,\tau) \quad (5.b)$$

$$C_p(\theta_\varepsilon) \approx C_p(\theta) + \frac{dC_p}{d\theta}\varepsilon\Delta\theta \quad (5.c)$$

$$K(\theta_\varepsilon) \approx K(\theta) + \frac{dK}{d\theta}\varepsilon\Delta\theta \quad (5.d)$$

The sensitivity problem is obtained by applying the following limiting process:

$$\lim_{\varepsilon \to 0} \frac{L_\varepsilon(Bi_\varepsilon) - L(Bi)}{\varepsilon} = 0 \quad (6)$$

where $L_\varepsilon(Bi_\varepsilon)$ and $L(Bi)$ are the operator forms of the direct problem, written for the perturbed and unperturbed quantities, respectively. The following problem results for the *sensitivity function* $\Delta\theta(X, Y, Z, \tau)$:

$$\frac{\partial}{\partial\tau}(C_p\Delta\theta) = \frac{\partial^2(K\Delta\theta)}{\partial X^2} + \frac{\partial^2(K\Delta\theta)}{\partial Y^2} + \frac{\partial^2(K\Delta\theta)}{\partial Z^2} \quad (7.a)$$

in -A/2<X<A/2, -B/2<Y<B/2, 0<Z<1, for τ>0

$$\frac{\partial(K\Delta\theta)}{\partial X} = 0 \quad \text{at } X=-A/2,\ X=A/2, \text{ for } \tau>0 \quad (7.b,c)$$

$$\frac{\partial(K\Delta\theta)}{\partial Y} = 0 \quad \text{at } Y=-B/2,\ Y=B/2, \text{ for } \tau>0 \quad (7.d,e)$$

$$\Delta\theta = 0 \quad \text{at } Z=0, \text{ for } \tau>0 \quad (7.f)$$

$$\frac{\partial(K\Delta\theta)}{\partial Z} + Bi\Delta\theta = -\Delta Bi\,\theta \quad \text{at } Z=1, \text{ for } \tau>0 \quad (7.g)$$

$$\Delta\theta = 0 \quad \text{for } \tau=0 \text{ in } -A/2<X<A/2,\ -B/2<Y<B/2,\ 0<Z<1 \quad (7.h)$$

ADJOINT PROBLEM

An adjoint problem for a Lagrange Multiplier comes into picture, because the temperature $\theta(X_s, Y_s, Z_s, \tau; Bi)$ appearing in the functional (4) needs to satisfy a constraint, given by the solution of the direct problem. In order to develop the adjoint problem, we multiply the differential equation (1.a) of the direct problem by the Lagrange Multiplier $\lambda(X, Y, Z, \tau)$, integrate over the time and space domains, and add the resulting expression to the functional given by Eq. (4). The following extended functional is obtained:

$$J[Bi] = \frac{1}{2}\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2}\int_{Z=0}^{1}\int_{\tau=0}^{\tau_f} \sum_{s=1}^{S}\left[\theta_S(\tau) - \mu_S(\tau)\right]^2 \delta(\mathbf{r}-\mathbf{r}_S)\, d\tau\, dZ\, dY\, dX +$$
$$\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2}\int_{Z=0}^{1}\int_{\tau=0}^{\tau_f}\left\{C_p\frac{\partial\theta}{\partial\tau} - \frac{\partial}{\partial X}\left(K\frac{\partial\theta}{\partial X}\right) - \frac{\partial}{\partial Y}\left(K\frac{\partial\theta}{\partial Y}\right) - \frac{\partial}{\partial Z}\left(K\frac{\partial\theta}{\partial Z}\right)\right\}\lambda(X,Y,Z,\tau)\, d\tau\, dZ\, dY\, dX \quad (8)$$

where $\delta(\ )$ is the Dirac delta function and $\mathbf{r}_S$ is the vector with the position of sensor s, i.e., $\mathbf{r}_S = (X_s, Y_s, Z_s)$.

An expression for the directional derivative of J[Bi] in the direction of the perturbation $\Delta Bi(X, Y, \tau)$ is obtained by applying the following limiting process:

$$D_{\Delta Bi} J\left[Bi\right] = \lim_{\varepsilon \to 0} \frac{J\left[Bi_{\varepsilon}\right] - J\left[Bi\right]}{\varepsilon} \tag{9}$$

where $J[Bi_{\varepsilon}]$ is the functional (8) written for the perturbed quantities given by Eqs. (5).

After performing some integrations by parts on the resulting expression for $D_{\Delta Bi}J[Bi]$ and applying the boundary and initial conditions of the sensitivity problem, we let the terms containing $\Delta\theta(X, Y, Z, \tau)$ to go to zero. The following *adjoint problem* is then obtained for the Lagrange Multiplier $\lambda(X, Y, Z, \tau)$:

$$-C_p\frac{\partial \lambda}{\partial \tau} - K\frac{\partial^2 \lambda}{\partial X^2} - K\frac{\partial^2 \lambda}{\partial Y^2} - K\frac{\partial^2 \lambda}{\partial Z^2} + \sum_{s=1}^{S}(\theta_s - \mu_s)\,\delta(\mathbf{r} - \mathbf{r}_s) = 0$$

in -A/2<X<A/2, -B/2<Y<B/2, 0<Z<1, for τ>0 (10.a)

$$\frac{\partial \lambda}{\partial X} = 0 \quad \text{at } X=-A/2,\ X=A/2, \text{ for } \tau>0 \tag{10.b,c}$$

$$\frac{\partial \lambda}{\partial Y} = 0 \quad \text{at } Y=-B/2,\ Y=B/2, \text{ for } \tau>0 \tag{10.d,e}$$

$$\lambda = 0 \quad \text{at } Z=0, \text{ for } \tau>0 \tag{10.f}$$

$$K\frac{\partial \lambda}{\partial Z} + Bi\,\lambda = 0 \quad \text{at } Z=1, \text{ for } \tau>0 \tag{10.g}$$

$$\lambda = 0 \quad \text{for } \tau=\tau_f \text{ in } -A/2<X<A/2,\ -B/2<Y<B/2,\ 0<Z<1 \tag{10.h}$$

GRADIENT EQUATION

In the process of obtaining the adjoint problem, the following integral term is left:

$$D_{\Delta Bi} J\left[Bi\right] = \int_{\tau=0}^{\tau_f}\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2} \lambda(X,Y,1,\tau)\, \theta(X,Y,1,\tau)\,\Delta Bi(X,Y,\tau)\; dY\; dX\; d\tau \tag{11}$$

By assuming that $Bi(X, Y, \tau)$ belongs to the Hilbert space of square integrable functions in the domain $(0, \tau_f)$ x (-A/2, A/2) x (-B/2, B/2), we can write

$$D_{\Delta Bi} J\left[Bi\right] = \int_{\tau=0}^{\tau_f}\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2} J'(X,Y,\tau)\,\Delta Bi(X,Y,\tau)\; dY\; dX\; d\tau \tag{12}$$

Therefore, by comparing Eqs. (11) and (12), we obtain the gradient equation for the functional as

$$J'(X,Y,\tau) = \lambda(X,Y,1,\tau)\,\theta(X,Y,1,\tau) \tag{13}$$

CONJUGATE GRADIENT METHOD OF MINIMIZATION

The iterative procedure of the conjugate gradient method (Jarny et al, 1991, Alifanov, 1994), as applied to the estimation of the unknown heat transfer coefficient, is given by

$$Bi^{k+1}(X,Y,\tau) = Bi^{k}(X,Y,\tau) - \beta^{k} d^{k}(X,Y,\tau) \tag{14.a}$$

where the superscript k denotes the number of iterations. The *direction of descent* is a conjugation of the gradient direction and of the previous direction of descent, given in the form

$$d^{k}(X,Y,\tau) = J'^{k}(X,Y,\tau) + \gamma^{k} d^{k-1}(X,Y,\tau) \tag{14.b}$$

The *conjugation coefficient* utilized here was obtained from the Fletcher-Reeves expression (Alifanov, 1994):

$$\gamma^{k} = \frac{\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2}\int_{\tau=0}^{\tau_f}\left[J'^{k}(X,Y,\tau)\right]^2 d\tau\; dY\; dX}{\int_{X=-A/2}^{A/2}\int_{Y=-B/2}^{B/2}\int_{\tau=0}^{\tau_f}\left[J'^{k-1}(X,Y,\tau)\right]^2 d\tau\; dY\; dX} \tag{14.c}$$

with γ^0=0 for k=1,2,…

The search step size β^k is obtained by minimizing the functional $J[Bi^{k+1}]$ given by Eq. (4) with respect to β^k. The following expression results:

$$\beta^{k} = \frac{\int_{\tau=0}^{\tau_f}\sum_{s=1}^{S}\left[\theta_s(\tau) - \mu_s(\tau)\right]\Delta\theta_s(d^k)\,d\tau}{\int_{\tau=0}^{\tau_f}\sum_{s=1}^{S}\left[\Delta\theta_s(d^k)\right]^2 d\tau} \tag{15}$$

where $\Delta\theta_s(d^k)$ is the solution of the sensitivity problem given by Eqs. (7), obtained by setting $\Delta Bi(X, Y, \tau) = d^k(X, Y, \tau)$.

STOPPING CRITERION

We stop the iterative procedure of the conjugate gradient method when the functional given by Eq. (4) becomes sufficiently small, that is,

$$J\left[Bi^{k+1}(X,Y,\tau)\right] < \varepsilon \qquad (16)$$

If the measurements are assumed to be free of experimental errors, we can specify ε as a relatively small number. However, actual measured data contain experimental errors, which will introduce oscillations in the inverse problem solution, as the estimated temperatures approach those measured. Such difficulty can be alleviated by utilizing the *Discrepancy Principle* (Alifanov, 1994) to stop the iterative process. Hence, we assume that the inverse problem solution is sufficiently accurate when the difference between estimated and measured temperatures is less than the standard deviation, σ, of the measurements. Thus, the value of the tolerance ε is obtained from Eq. (4) as

$$\varepsilon = \frac{1}{2} \Sigma \sigma^2 \tau_f \qquad (17)$$

COMPUTATIONAL ALGORITHM

Suppose an estimate $Bi^k(X, Y, \tau)$ is available for the unknown heat transfer coefficient $Bi(X, Y, \tau)$ at iteration k. Thus:

STEP 1: Solve the direct problem given by Eqs. (1) to obtain the estimated temperatures $\theta(X, Y, Z, \tau)$;

STEP 2: Check the stopping criterion given by Eq. (16) Continue if not satisfied;

STEP 3: Solve the adjoint problem given by Eqs. (10) to obtain the Lagrange Multiplier $\lambda(X, Y, Z, \tau)$;

STEP 4: Compute the gradient of the functional $J'^k(X, Y, \tau)$ from Eq. (13);

STEP 5: Compute the conjugation coefficient γ^k from Eq. (14.c) and then the direction of descent $d^k(X, Y, \tau)$ from Eq. (14.b);

STEP 6: Solve the sensitivity problem given by Eqs. (7) to obtain $\Delta\theta(X, Y, Z, \tau)$ by setting $\Delta Bi(X, Y, \tau) = d^k(X, Y, \tau)$;

STEP 7: Compute the search step size β^k from Eq. (15);

STEP 8: Compute the new estimate $Bi^{k+1}(X, Y, \tau)$ from Eq. (14.a) and return to step 1.

RESULTS AND DISCUSSION

As a test-problem, we consider here the cooling of a steel plate with dimensions a=0.3m, b=1m and c=0.1m, initially at a uniform temperature T_0=1000 °C. For times t>0, the top surface of the plate is cooled by an air-mist spray, and the temperature of the cooling fluid is supposed to be T_∞=25 °C. The bottom surface of the plate is supposed to be maintained at the constant temperature of 25 °C. The physical properties of the plate are assumed constant and their values are taken as (Ozisik, 1993): ρ^*=7753 Kg/m³, K_0^*=36 W/m°C and C_0^*=0.486 KJ/Kg°C. The final experimental time is taken as t_f = 1.7 min. The dimensionless variables associated with the values above are: A=3, B=10, τ_f = 0.1, φ=0 and ϕ=1.

We use simulated experimental data in order to assess the accuracy of the conjugate gradient method with adjoint equation, as applied to the estimation of Bi(X , Y, τ). For the solution of the present inverse problem, transient measurements of multiple sensors are required, in order to recover the spatial and time dependencies of the unknown function. The simulated experimental data were obtained from the solution of the direct problem for an *a priori* assumed functional form for Bi(X, Y, τ). The solution of the direct problem provides the *exact* measurements, μ_{exa}. Measurements containing errors are simulated by adding a random error term to μ_{exa} in the form

$$\mu = \mu_{exa} + \omega\sigma \qquad (18)$$

where ω is a random variable with normal distribution, zero mean and unitary standard-deviation. It is obtained with the subroutine DRRNOR of the IMSL (1996). The value of σ is the standard deviation of the measurement errors, which is assumed to be constant.

In order to generate the simulated measurements, the function Bi(X, Y, τ) was written in the following form:

$$Bi = 6 f_\tau(\tau) f_X(X) f_Y(Y) \qquad (19)$$

We note that the constant 6 appearing in Eq. (19) gives a heat transfer coefficient of approximately 2200 W/m²°C for $f_\tau(\tau) = f_X(X) = f_Y(Y) = 1$. Values for the heat transfer coefficient were reported in the range 400 to 6000 W/m²°C, depending on the spray operating conditions and plate surface temperature (Mizikar, 1970, Hibbins and Brimacombe, 1984, Lally et al, 1990).

Different functional forms were tested for $f_\tau(\tau)$, $f_X(X)$ and $f_Y(Y)$, including:

$$f_\tau(\tau) = 1 \qquad (20.a)$$

$$f_\tau(\tau) = \begin{cases} 1 & \text{for } \tau \le 0.0333 \text{ or } \tau \ge 0.0667 \\ 2 & \text{for } 0.0333 < \tau < 0.0667 \end{cases} \qquad (20.b)$$

$$f_\tau(\tau) = \begin{cases} 1 & \text{for } \tau \le 0.0333 \text{ or } \tau \ge 0.0667 \\ -1 + 60\tau & \text{for } 0.0333 < \tau \le 0.05 \\ 5 - 60\tau & \text{for } 0.05 < \tau < 0.0667 \end{cases} \qquad (20.c)$$

$$f_X(X) = e^{-X^2} \tag{21}$$

$$f_Y(Y) = 1 \tag{22.a}$$

$$f_Y(Y) = e^{-Y^2} \tag{22.b}$$

Equations (20.b,c) were chosen because they represent functions containing discontinuities and sharp corners in time, respectively. Such kinds of functions are the most difficult to be recovered by inverse analysis. Equation (20.a) was used to test the method for a function with no time dependence. Equations (21, 22) were chosen because an exponential decay for Bi(X, Y, τ), with the distance from the point where the spray nozzle is located, has been reported (Lally et al, 1990). Therefore, a spatial variation for Bi(X, Y, τ) obtained from Eqs. (21) and (22.a) would correspond to a row of spray nozzles located above the Y axis. Similarly, the combination of Eqs. (21) and (22.b) would correspond to a single spray nozzle located above X=Y=0.

Table I - RMS error for different number of sensors

Bi(X, Y, τ)	σ	Number of Sensors 12	16	20
$6\ e^{-X^2}$	0	0.719	0.073	0.050
	0.01	0.723	0.307	0.263
$6\ e^{-X^2} e^{-Y^2}$	0	1.789	0.103	0.081
	0.01	1.789	0.275	0.314

Table II - X and Y positions for sensor location

Number of Sensors	X	Y
12	0.0; 0.75; 1.50	0.0; 1.5; 3.5; 5.0
16	0.0; 0.45; 1.05; 1.50	0.0; 1.5; 3.5; 5.0
20	0.0; 0.45; 1.05; 1.50	0.0; 1.0; 2.5;4.0; 5.0

Due to the symmetry with respect to the X and Y axes of the spatial variations tested, Eqs. (21, 22), we solved the direct, sensitivity and adjoint problems in the reduced domain $0 \le X \le A/2$, $0 \le Y \le B/2$ and $0 \le Z \le 1$. These problems were solved by finite differences by discretizing the spatial domain with 11x11x41 points in the X x Y x Z directions, respectively, and with 160 time-steps. Such number of points were chosen by comparing the finite-difference solution for the direct problem with a known analytical solution involving a constant Bi (Ozisik, 1993). The agreement between the two solutions was better than 0.2%. We have also performed a grid convergence analysis for cases involving Bi(X, Y, τ) obtained with different combinations of Eqs. (20-22). The results obtained with the discretization above were within a maximum difference of 0.66%, with respect to those obtained by doubling the number of points in each spatial direction or in the time domain.

We examine here two different standard-deviations for the measurement errors: σ=0 (errorless measurements) and σ=0.01. The standard deviation σ=0.01 is characteristic of the measurement system to be used in the actual experiments and yields errors of the same order of those reported by Hibbins and Brimacombe (1984).

Table I presents the RMS errors obtained with 12, 16 and 20 sensors, for two spatial variations of Bi(X, Y, τ). The function was supposed to be constant in time and 160 transient measurements per sensor were used in the inverse analysis. The sensors were located at Z=0.95, which corresponds to 5 mm below the spray cooled surface, and in a grid formed by the X and Y positions shown in Table II. The RMS error is defined here as:

$$e_{RMS} = \sqrt{\frac{1}{I} \sum_{i=1}^{I} \left[Bi_{est}(X_i, Y_i, t_i) - Bi_{ex}(X_i, Y_i, t_i) \right]^2} \qquad (23)$$

where I is the total number of measurements. The subscripts "est" and "ex" refer to the estimated and exact dimensionless heat transfer coefficient, respectively.

Table I shows that there is a large reduction on the RMS error when 16 sensors are used instead of 12. However, the reduction on the RMS error is not significant when the number of sensors is increased to 20. Hence, an examination of Table I reveals that the transient readings of a minimum of 16 sensors should be used in order to recover the spatial variation of the heat transfer coefficient.

Table III presents a comparison of the inverse problem solution, obtained with 160 transient measurements of 16 sensors, located at Z=0.95 and at Z=0.975, corresponding to 5 mm and 2.5 mm below the surface, respectively. A step variation in time for Bi(X, Y, τ), Eq. (20.b), was used for this comparison. As expected, we note in Table III a reduction on the RMS error by locating the sensors closer to the surface with the unknown heat transfer coefficient, since less information is lost due to the diffusive character of the problem.

Figures 2 and 3 present the exact and estimated functions for the same time and spatial variations considered in Table III. The results presented in these figures were obtained with measurements containing random errors (σ=0.01) of 16 sensors located at Z=0.975. An examination of Eqs. (10.h) and (13) reveals that the gradient equation is null at the final time. Hence, the initial guess used for $Bi(X, Y, \tau_f)$ is not changed by the iterative procedure of the conjugate gradient method, and oscillations on the estimated function are observed in the neighborhood of τ_f. In order to avoid such difficulties, the initial guess was taken as the exact solution for the final time, which was assumed known *a priori*.

We note in Fig. 2 that accurate results are obtained for the spatial variation given by $6\ e^{-X^2}$. The method is able to correctly predict the exponential decay in the X direction, while the solution is independent of Y. Figure 2 shows that the estimated function follows the exact step variation in time quite closely, even for X=0, where the largest discontinuity takes place. Note also that the method is able to recover small variations, such as for X=1.5, reasonably well.

Figures 3 show the results obtained for a function involving a spatial variation with X and Y in the form $6\ e^{-X^2} e^{-Y^2}$, given by Eqs. (21) and (22.b). We note that the exponential decays in X and Y are correctly predicted. The estimation of the step variation in time is also quite good, except at the position X=0, Y=0, where the largest discontinuity takes place.

Table III - RMS error for different sensor positions

Bi(X, Y, τ)		σ	Sensor Positions Z=0.95	Z=0.975
$6\ e^{-X^2}$	eq. (21.b)	0.0	0.959	0.346
		0.01	0.979	0.627
$6\ e^{-X^2} e^{-Y^2}$	eq. (21.b)	0.0	0.582	0.398
		0.01	0.525	0.479

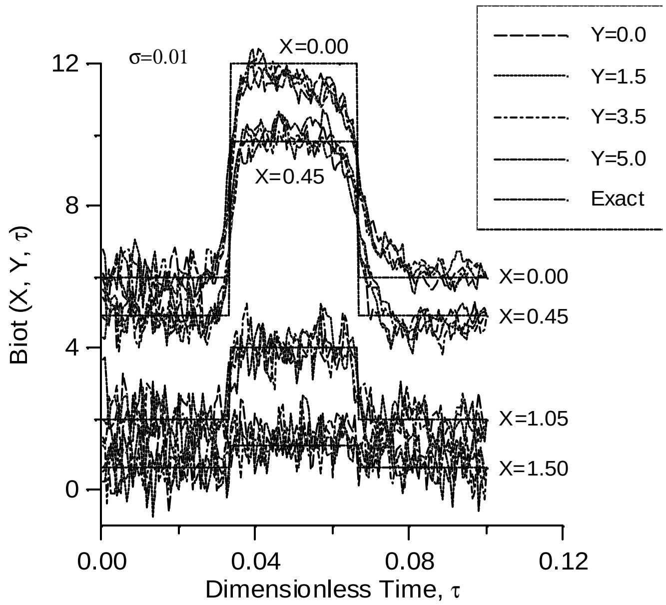

Figure 2 - Solution for a step variation in time and exponential decay in X

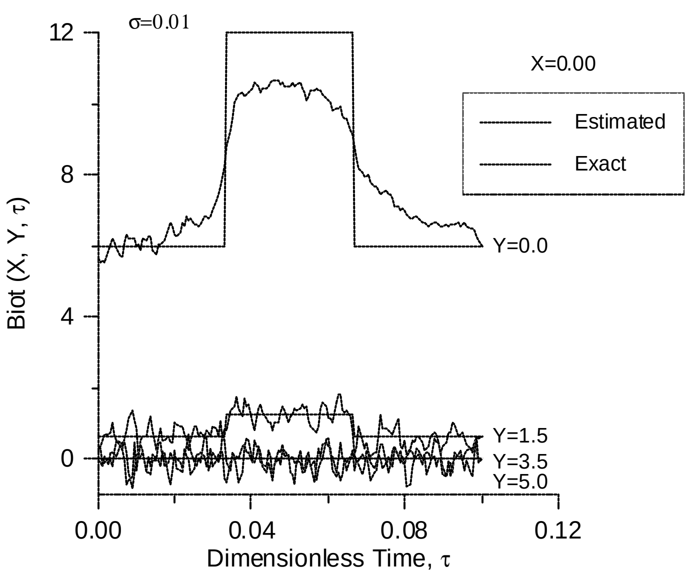

Figure 3.a - Solution for a step variation in time and exponential decays in X and Y for X=0

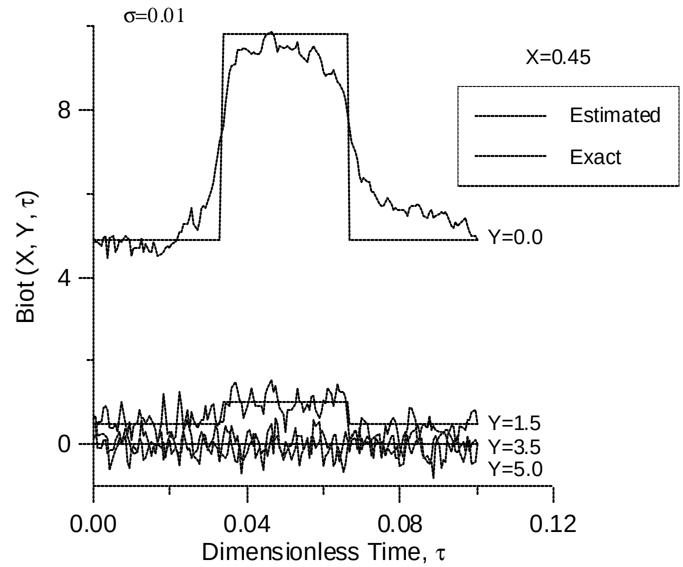

Figure 3.b - Solution for a step variation in time and exponential decays in X and Y for X=0.45

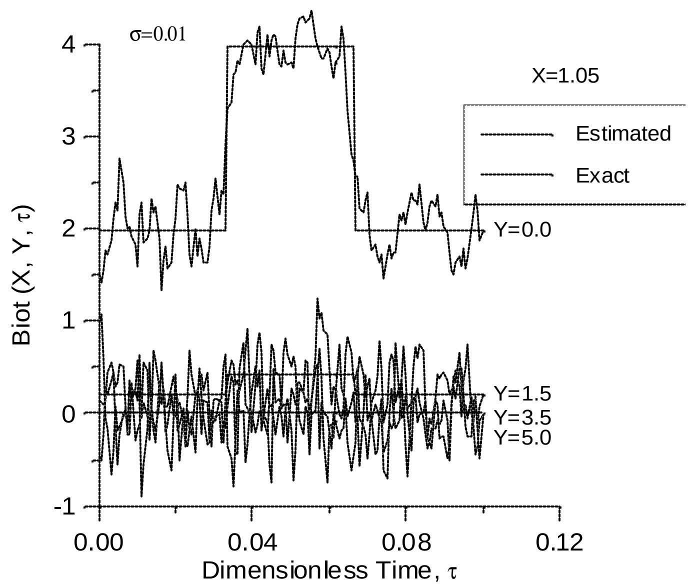

Figure 3.c - Solution for a step variation in time and exponential decays in X and Y for X=1.05

Figure 4 shows the results obtained for conditions similar to those of Fig. 2, but for a triangular variation in time. We note that the general characters of the time and spatial variations of Bi (X, Y, τ) are correctly recovered, although some smoothness is noticed in the peak at τ=0.05, for X=0. A function with exponential decays in X and Y, Eqs. (21) and (22.b), was also tested with a triangular variation in time. The results obtained were similar to those shown in Figs. 3 and, therefore, are omitted here for the sake of brevity.

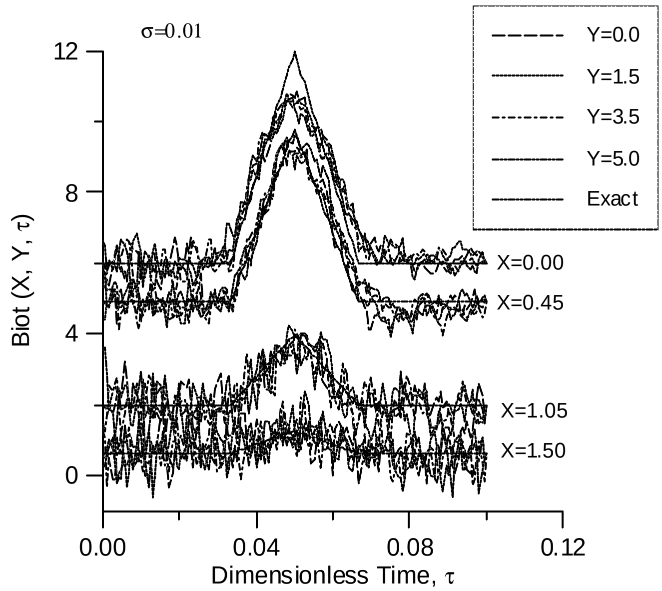

Figure 4 - Solution for a triangular variation in time and exponential decay in X

CONCLUSIONS

In this paper, we solved the inverse problem of estimating the heat transfer coefficient at the surface of a plate. Such heat transfer coefficient was supposed to vary in time and spatially over the plate surface. The conjugate gradient method with adjoint equation was applied as a function estimation approach.

Results obtained with simulated measurements, for functions involving exponential decays in the spatial domain, with step and triangular variations in the time domain, revealed that the method is stable with respect to measurement errors and capable of providing accurate estimates for the unknown heat transfer coefficient. Any dependence of the heat transfer coefficient with temperature can also be recovered with the present approach, since the local surface temperatures of the plate are estimated as part of the solution of the inverse problem.

The use of simulated measured data also allowed for the design of an experimental apparatus, with respect to the number and location of the temperature sensors. Such experimental apparatus is currently under construction and will be applied to the estimation of the heat transfer coefficient of air-mist sprays, utilized in the cooling of continuously cast slabs.

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