A FUNCTION ESTIMATION APPROACH FOR THE IDENTIFICATION OF THE TRANSIENT INLET PROFILE IN PARALLEL PLATE CHANNELS

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ABSTRACT

This paper deals with the use of the conjugate gradient method of function estimation for the identification of the inlet temperature profile, also supposed to vary with time, in a parallel plate channel. The fluid flow is assumed to be laminar and hydrodynamically developed. The channel walls are subjected to a constant temperature boundary condition. Temperature measurements taken downstream are used in the inverse analysis. The accuracy of the present solution approach is examined by using simulated measurements containing random errors, for strict cases involving functional forms with discontinuities or sharp-corners for the inlet temperature profile.

KEYWORDS

Conjugate gradient method, function estimation, adjoint problem, forced convection, laminar flow, transient inlet profile

INTRODUCTION

Heat conduction was the first heat transfer mode to be addressed in the inverse problem literature. This is probably because of the interest of researchers involved with the space program in the 50's and 60's, aiming at the accurate estimation of thermal properties of heat shields, as well as of the heat flux on the surface of re-entrant vehicles. Some heuristic methods of solution for inverse problems, which were based more on pure intuition than on mathematical formality, were developed in the 50's. Later in the 60's and 70's, most of the methods, which are in common use nowadays, were formalized in terms of their capabilities to treat ill-posed unstable problems. The basis of such formal methods resides on the idea of reformulating the inverse problem in terms of an approximate well-posed problem, by utilizing some kind of regularization (stabilization) technique. In this sense, it is recognized here the pioneering works of scientists who found different forms of overcoming the instabilities of inverse problems, including A. N. Tikhonov[1-3], O. M. Alifanov[4-6] and J. V. Beck[7-9].

The interest on the solution of inverse heat convection problems is more recent than on the solution of inverse heat conduction problems. To the best of the authors' knowledge, the first article dealing with an inverse heat convection problem is the one due to Moutsoglou, from 1989 [10]. Afterwards, several other articles involving inverse convection appeared in the literature[11-23]. In their majority, those papers dealt with forced convection inside tubes or channels [11-19,21,23], with unknown wall heat flux [11,12,14-16,19,21,23] or unknown inlet condition [13,17,18]. With respect to the unknown inlet condition, either a uniform transient inlet temperature [17], or a steady-state inlet temperature profile (varying across the channel), have been considered as the unknown quantity [13,18].

In this paper, we present the solution of the inverse problem of estimating the temperature inlet profile, also supposed to vary with time, in a parallel plate channel. As the solution technique, we apply the conjugate gradient method of function estimation [4-6,12,14-19,24], by assuming that no information is available regarding the time and spatial variations of the unknown function. Simulated temperature measurements taken inside the channel are used in the inverse analysis, in order to address the accuracy of the present solution technique, as well as to design the experiment with respect to the number and locations of temperature sensors. Basic steps of the conjugate gradient method of function estimation include: (i) The Direct Problem, (ii) The Inverse Problem, (iii) The Sensitivity Problem, (iv) The Adjoint Problem, (v) The Gradient Equation, (vi) The Iterative Procedure, (vii) The Stopping Criterion and (viii) The Computational Algorithm. Details of such steps, as applied to the present inverse problem, are described next.

DIRECT PROBLEM

The physical problem considered in this work involves the laminar hydrodynamically-developed forced convection of a Newtonian fluid in a parallel plate channel. The inlet temperature, f(y,t), may vary with time as well as across the channel. The channel walls, separated by a distance 2h, are assumed to be maintained at a constant temperature T_i, equal to the initial fluid temperature inside the channel. The formulation of such physical problem is given in dimensionless form as:

$$\frac{\partial^2 \Theta}{\partial Y^2} = \frac{\partial \Theta}{\partial \tau} + U(Y)\frac{\partial \Theta}{\partial X} \qquad \text{in } 0 < Y < 1,\ X > 0;\ \text{for } \tau > 0 \qquad (1.a)$$

$$\Theta = 0 \qquad \text{at } Y=0,\ X > 0;\ \text{for } \tau > 0 \qquad (1.b)$$

$$\Theta = 0 \qquad \text{at } Y=1,\ X > 0;\ \text{for } \tau > 0 \qquad (1.c)$$

$$\Theta = F(Y,\tau) \qquad \text{at } X=0,\ 0 < Y < 1;\ \text{for } \tau > 0 \qquad (1.d)$$

$$\Theta = 0 \qquad \text{For } \tau = 0;\ \text{in } 0 < Y < 1,\ X > 0 \qquad (1.e)$$

where the following dimensionless groups were defined

$$\tau = \frac{\alpha\, t}{4\, h^2} \tag{2.a}$$

$$Y = \frac{y}{2\, h} \tag{2.b}$$

$$X = \frac{\alpha\, x}{u_m\, 4\, h^2} \tag{2.c}$$

$$F(Y, \tau) = \frac{f(y, t) - T_i}{T_i} \tag{2.d}$$

and the dimensionless velocity profile is given by:

$$U(Y) = 6\, Y\, (1 - Y) \tag{2.e}$$

The *direct problem* is concerned with the determination of the temperature distribution inside the channel, from the knowledge of the velocity profile (2.e) and from the inlet condition $F(Y,\tau)$.

INVERSE PROBLEM

The *inverse problem* under picture in this paper is concerned with the estimation of the time and spatial variations of the inlet temperature, $F(Y,\tau)$, by using temperature measurements taken downstream. The inverse problem is reformulated as a minimization problem involving the following objective functional:

$$S[F(Y,t)] = \int_{t=0}^{t_f} \sum_{n=1}^{N} \left\{ Z_n(t) - \Theta[\mathbf{r}_n, t; F(Y,t)] \right\}^2 dt \tag{3}$$

where N is the number of sensors used in the analysis, $Z_n(\tau)$ are the measured temperatures at the position $\mathbf{r}_n$, while $\Theta[\mathbf{r}_n,\tau;F(Y,\tau)]$ are the estimated temperatures at the measurement positions.

The minimization of the objective functional (3) is obtained through the conjugate gradient method [4-6,12,14-19,24]. Two auxiliary problems, known as the *sensitivity and adjoint problems*, are required for the implementation of the iterative procedure of such a method, as described next. For further details on the derivation of these problems the reader is referred to references [4-6,12,14-19,24].

SENSITIVITY PROBLEM

In order to develop the sensitivity problem, we assume that the temperature $\Theta(X,Y,\tau)$ undergoes a variation $\Delta\Theta(X,Y,\tau)$, when the inlet temperature undergoes a variation $\Delta F(Y,\tau)$. By substituting into the direct problem (1) $\Theta(X,Y,\tau)$ by $[\Theta(X,Y,\tau)+\Delta\Theta(X,Y,\tau)]$ and $F(Y,\tau)$ by $[F(Y,\tau)+\Delta F(Y,\tau)]$, and then subtracting from the resulting equations the original direct problem, we obtain the following *sensitivity problem* for the *sensitivity function* $\Delta\Theta(X,Y,\tau)$ [4-6,12,14-19,24]:

$$\frac{\partial^2 \Delta\Theta}{\partial Y^2} = \frac{\partial \Delta\Theta}{\partial \tau} + U(Y)\frac{\partial \Delta\Theta}{\partial X} \quad \text{in } 0 < Y < 1,\ X > 0; \text{ for } \tau > 0 \tag{4.a}$$

$$\Delta\Theta = 0 \quad \text{at } Y=0,\ X > 0; \text{ for } \tau > 0 \tag{4.b}$$

$$\Delta\Theta = 0 \quad \text{at } Y=1,\ X > 0; \text{ for } \tau > 0 \tag{4.c}$$

$$\Delta\Theta = \Delta F(Y,\tau) \quad \text{at } X=0,\ 0 < Y < 1; \text{ for } \tau > 0 \tag{4.d}$$

$$\Delta\Theta = 0 \quad \text{for } \tau = 0; \text{ in } 0 < Y < 1,\ X > 0 \tag{4.e}$$

ADJOINT PROBLEM

The adjoint problem is obtained by multiplying equation (1.a) by the *Lagrange Multiplier* $\lambda(X,Y,\tau)$, integrating the resulting expression over the time and space domains and adding the result to the functional given by equation (3). We obtain:

$$S[F(Y,t)] = \int_{t=0}^{t_f} \sum_{n=1}^{N} \{Z_n(t) - \Theta[\mathbf{r}_n, t; F(Y,t)]\}^2 \, dt + \int_{Y=0}^{1}\int_{X=0}^{L}\int_{t=0}^{t_f} l(X,Y,t) \left\{ \frac{\partial^2 \Theta(X,Y,t)}{\partial Y^2} - \frac{\partial \Theta(X,Y,t)}{\partial t} - U(Y)\frac{\partial \Theta(X,Y,t)}{\partial X} \right\} dt\, dX\, dY \tag{5}$$

We now perturb $F(Y,\tau)$ by $\Delta F(Y,\tau)$ and $\Theta(X,Y,\tau)$ by $\Delta\Theta(X,Y,\tau)$ in equation (5) and subtract equation (5) from the resulting expression to get the variation $\Delta S[F(Y,\tau)]$ of the functional $S[F(Y,\tau)]$. By employing integration by parts, utilizing the initial and boundary conditions of the sensitivity problem and also requiring that the coefficients of $\Delta\Theta(X,Y,\tau)$ in the resulting equation should vanish, the following *adjoint problem* is obtained [4-6,12,14-19,24]:

$$\frac{\partial^2 l}{\partial Y^2} + \frac{\partial l}{\partial t} + U(Y)\frac{\partial l}{\partial X} + 2\sum_{n=1}^{N} \{\Theta[\mathbf{r}, t; F(Y,t)] - Z_n(t)\} d(\mathbf{r} - \mathbf{r}_n) = 0 \quad \text{in } 0 < Y < 1,\ 0 < X < L; \text{ for } \tau > 0 \tag{6.a}$$

where $d(\cdot)$ is the Dirac delta function, and the boundary conditions become

$$l = 0 \quad \text{at } Y=0,\ 0 < X < L; \text{ for } \tau > 0 \tag{6.b}$$

$$l = 0 \quad \text{at } Y=1,\ 0 < X < L; \text{ for } \tau > 0 \tag{6.c}$$

$$l = 0 \quad \text{at } X=L,\ 0 < Y < 1; \text{ for } \tau > 0 \tag{6.d}$$

$$l = 0 \quad \text{for } \tau = \tau_f; \text{ in } 0 < Y < 1,\ 0 < X < L \tag{6.e}$$

where L is the channel dimensionless length.

GRADIENT EQUATION

In the process of obtaining the adjoint problem, the variation of the functional reduces to

$$\Delta S[F(Y,t)] = \int_{t=0}^{t_f} \int_{Y=0}^{1} U(Y)\, l(0,Y,t)\, \Delta F(Y,t)\, dY\, dt \tag{7.a}$$

By assuming that the function F(Y,τ) belongs to the space of square integrable functions in $0 < \tau < \tau_f$, $0 < Y < 1$, we can write

$$\Delta S[F(Y,t)] = \int_{t=0}^{t_f} \int_{Y=0}^{1} \nabla S[F(Y,t)]\, \Delta F(Y,t)\, dY\, dt \tag{7.b}$$

Hence, by comparing equations (7.a) and (7.b), we can obtain the gradient equation for the functional as:

$$\nabla S[F(Y,t)] = U(Y)\, l(0,Y,t) \tag{8}$$

ITERATIVE PROCEDURE

The iterative procedure of the conjugate gradient method, as applied to the estimation of the function F(Y,τ) is given as [4-6,12,14-19,24]:

$$F^{k+1}(Y,t) = F^{k}(Y,t) - b^{k}\, d^{k}(Y,t) \tag{9.a}$$

where k is the number of iterations. The direction of descent $d^k(Y,\tau)$ is obtained from

$$d^{k}(Y,\tau) = \nabla S\left[F^{k}(Y,\tau)\right] + \gamma^{k}\, d^{k-1}(Y,\tau) \tag{9.b}$$

The conjugation coefficient γ^k can be obtained from the Fletcher-Reeves expression as

$$\gamma^{k} = \frac{\int_{Y=0}^{1}\int_{\tau=0}^{\tau_f} \left\{\nabla S\left[F^{k}(Y,\tau)\right]\right\}^2 d\tau\, dY}{\int_{Y=0}^{1}\int_{\tau=0}^{\tau_f} \left\{\nabla S\left[F^{k-1}(Y,\tau)\right]\right\}^2 d\tau\, dY} \qquad \text{for } k=1,2,\ldots \tag{9.c}$$

with $\gamma^0 = 0$ for k = 0

The search step size β^k is obtained by minimizing $S[F^{k+1}(Y,\tau)]$ with respect to β^k. The following expression results

$$b^k = \frac{\int_{t=0}^{t_f} \sum_{n=1}^{N} \left\{ \Theta\left[\mathbf{r}_n, t; F^k(Y,t)\right] - Z_n(t) \right\} \Delta\Theta\left[\mathbf{r}_n, t; d^k(Y,t)\right] dt}{\int_{t=0}^{t_f} \sum_{n=1}^{N} \left\{ \Delta\Theta\left[\mathbf{r}_n, t; d^k(Y,t)\right] \right\}^2 dt} \tag{9.d}$$

where $\Delta\Theta[\mathbf{r}_n,\tau;d^k(Y,\tau)]$ is the solution of the sensitivity problem (4), obtained by setting $\Delta F(Y,\tau)=d^k(Y,\tau)$.

STOPPING CRITERION

The iterative procedure of the conjugate gradient method does not possess by itself the regularization property required to obtain stable solutions for inverse problems, such as the one under picture in this paper. However, the use of the stopping criterion based on the *Discrepancy Principle* gives the conjugate gradient method an iterative regularization character [4-6]. In this case, the stopping criterion is given by

$$S\left[F(Y,t)\right] < e \tag{10}$$

where $S[F(Y,\tau)]$ is computed with equation (3). The tolerance ε is chosen so that smooth solutions are obtained with measurements containing random errors. It is assumed that the solution is sufficiently accurate when

$$\left| Z_n(t) - \Theta\left[\mathbf{r}_n, t; F(Y,t)\right] \right| \approx s \tag{11}$$

where σ is the constant standard deviation of the measurement errors.

Thus, ε is obtained from equation (3) as

$$e = N s^2 t_f \tag{12}$$

For cases involving errorless measurements, e can be specified *a priori* as a sufficiently small number, if the sensors are appropriately located. For those cases involving measurements with unknown standard deviation, an alternative approach based on an additional measurement can be used, as described in [6,24].

COMPUTATIONAL ALGORITHM

Suppose an initial guess $F^0(Y,\tau)$ is available for the function $F(Y,\tau)$. Set $k = 0$ and then:

Step 1. Solve the direct problem (1) and compute $\Theta(X,Y,\tau)$, based on $F^k(Y,\tau)$.
Step 2. Check the stopping criterion (10). Continue if not satisfied.
Step 3. Knowing $\Theta(X,Y,\tau)$ and measured temperatures $Z_n(\tau)$, solve the adjoint problem (6) and compute $\lambda(0,Y,\tau)$.
Step 4. Knowing $\lambda(0,Y,\tau)$, compute $\nabla S[F^k(Y,\tau)]$ from equation (8).

Step 5. Knowing the gradient $\nabla S[F^k(Y,\tau)]$, compute γ^k from equation (9.c).
Step 6. Set $\Delta F(Y,\tau)=d^k(Y,\tau)$ and solve the sensitivity problem (4) to obtain $\Delta\Theta[\mathbf{r}_n,\tau;d^k(Y,\tau)]$.
Step 7. Knowing $\Delta\Theta[\mathbf{r}_n,\tau;d^k(Y,\tau)]$, compute the search step size β^k from equation (9.d).
Step 8. Knowing the search step size β^k and the direction of descent $d^k(Y,\tau)$, compute the new estimate $F^{k+1}(Y,\tau)$ from equation (9.a), and return to step 1.

RESULTS AND DISCUSSIONS

In order to illustrate the accuracy of the present function estimation approach, we used simulated measurements containing random errors, normally distributed, with zero mean and constant standard-deviation (σ). Such simulated measurements were obtained by adding a random noise to the solution of the direct problem for an *a priori* established functional form for the inlet profile $F(Y,\tau)$. For the results presented below, the inlet profile was taken as $F(Y,\tau)=F_Y(Y)F_\tau(\tau)$. Functional forms containing discontinuities and sharp-corners were examined for $F_Y(Y)$ and $F_\tau(\tau)$, because they represent the most difficult functions to be recovered by inverse analysis. Let us consider a test-case with $F_Y(Y)$ and $F_\tau(\tau)$ taken, respectively, in the form:

$$F_Y(Y)=\begin{cases}1 & \text{for} \quad 0<Y\le 0.3 \quad \text{and} \quad 0.7\le Y<1 \\ 2 & \text{for} \quad 0.3<Y<0.7\end{cases} \tag{13.a}$$

$$F_t(Y)=\begin{cases}1 & \text{for} \quad 0<t\le 0.0006 \quad \text{and} \quad 0.0014\le t<0.0020 \\ 2500\,t-0.5 & \text{for} \quad 0.0006<t\le 0.0010 \\ -2500\,t+4.5 & \text{for} \quad 0.0010<t<0.0014\end{cases} \tag{13.b}$$

The values selected for the dimensionless variables correspond to a physical case involving the flow of water, initially at 20ºC, flowing at a mean-velocity of 0.01m/s in a channel 0.05m high and 0.36m long. The final physical time corresponds to 35s. For the results presented below, we assumed available 181 temperature readings per sensor, which corresponds to one measurement taken every 0.19s. The direct, sensitivity and adjoint problems were solved by finite-differences, by using an upwind discretization for the convection terms.

By examining equations (6.b,c,e) and (8), we note that the gradient equation is null at the final time, as well as at the channel walls. As a result, the initial guess used for the unknown function at such time and locations is not changed by the iterative procedure of the conjugate gradient method. Therefore, instabilities on the estimated function are expected in the neighborhood of such points. One approach to overcome such instabilities at τ_f is to consider a final time larger than of interest for the problem [24]. In the present case we used $\tau_f=0.0024$ for the inverse analysis and increased the number of measurements in time accordingly; but only the results in the domain of interest, i.e., $0 < \tau < 0.002$ are presented below. Also, sensors were not located at the channel walls, because no useful information would be recovered from the measurements taken at such positions.

Figures 1.a,b present the results for the spatial and time variations of $F(Y,\tau)$, respectively, obtained with measurements containing random errors of standard deviation $\sigma=0.01Z_{max}$, where Z_{max} is the maximum measured temperature. An initial guess of $F(Y,\tau)=0$ was used for the iterative procedure of the conjugate gradient method. Eighteen sensors uniformly distributed across the channel section, at the longitudinal position $X=3.9\text{x}10^{-4}$, were used in the analysis. Such longitudinal position corresponds to 19% of the channel length ($L=2.05x10^{-3}$); hence the sensors were located sufficiently far from the channel inlet. Figures 1.a,b show that very accurate estimates can be obtained for the spatial and time variations of $F(Y,\tau)$, with 18 sensors located at $X=3.9x10^{-4}$, for different times and transversal positions. As expected, the estimation of the time variation of the function was much more accurate than of the spatial variation, because the time frequency of measurements was ten times larger than the spatial distribution of the sensors across the channel. Figure 1.a clearly shows the effect of the null gradient at the channel walls, which caused deviations from the exact solutions at the neighborhood of Y=0 and Y=1. However, note that the effect of the null gradient at the final time is very little noticeable in Fig. 1.b. Such is the case because a final time 20% larger than that of interest was used in the analysis, as described above.

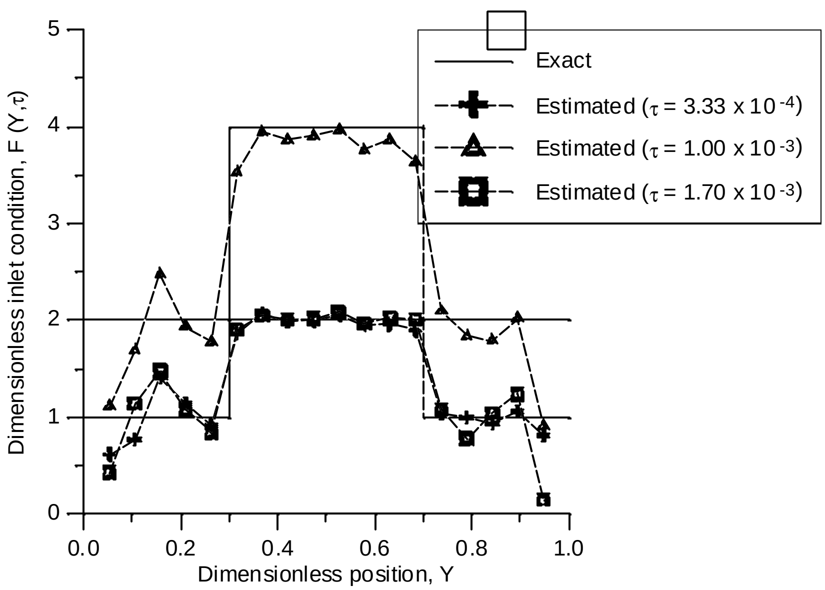

Fig. 1.a. Step variation of $F(Y,\tau)$ across the channel given by eq. (13.a).

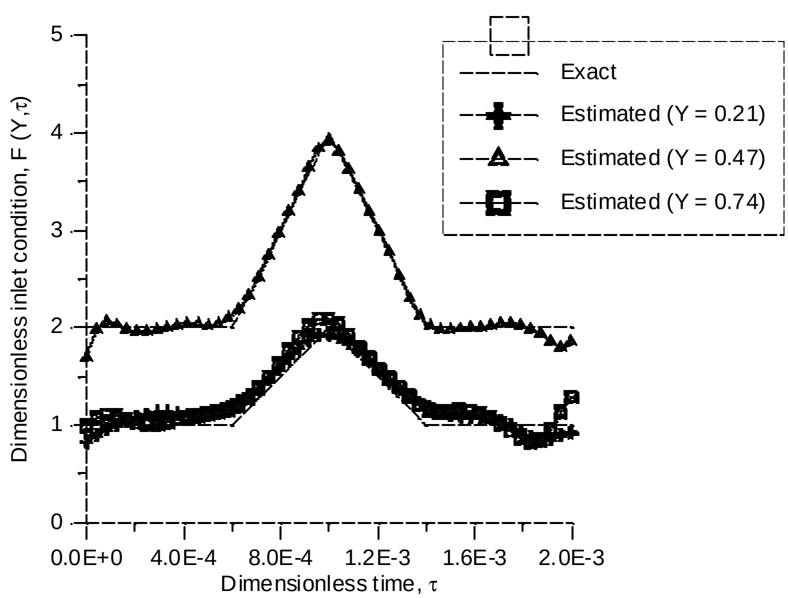

Fig. 1.b. Triangular variation of $F(Y,\tau)$ in time given by eq. (13.b).

The effects on the inverse problem solution of the longitudinal position and of the number of sensors are now examined. Table 1 illustrates the RMS errors for different test-cases, obtained with the same experimental conditions of the results shown in Figs. 1.a,b, except for the number and longitudinal position of the sensors. Note that test-case 2 corresponds to the one illustrated in Figs. 1.a,b. As expected, the RMS error decreased when the sensors were located closer to the channel inlet and when more sensors were used in the analysis. In fact, completely unrealistic spatial distributions were estimated with only 8 sensors, even when they were located very close to the channel inlet at $X=1.3\times10^{-4}$. On the other hand, the time variations were quite reasonably estimated, even for such low number of sensors. The RMS error is defined as

$$e_{RMS} = \frac{1}{NM}\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[F_{ex}\left(Y_n, t_m\right) - F_{est}\left(Y_n, t_m\right)\right]^2} \tag{14}$$

where M is the number of transient measurements taken per sensor, while the subscripts "ex " and "est" indicate exact and estimated quantities, respectively.

Table 1. RMS errors for different number and longitudinal position for the sensors

Test-case	**Number of Sensors**	**Longitudinal location**	e_{RMS}
1	18	1.3×10^{-4}	0.20
2	18	3.9×10^{-4}	0.27
3	18	5.2×10^{-4}	0.34
4	8	1.3×10^{-4}	0.56

CONCLUSIONS

In this paper we applied the conjugate gradient method for the identification of the transient inlet profile in a parallel plate channel, by using a function estimation approach, where no information is assumed available regarding the functional form of the unknown.

Results obtained with simulated temperature measurements reveal that quite accurate estimates can be obtained for the time and spatial variations of the unknown function, if the proper number of sensors is used and if they are suitably located inside the channel. The use of a final time larger than that of interest can reduce the effects of the null-gradient and improve the accuracy of the estimated function.

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