Inverse Convection Problems in Irregular Geometries

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Abstract

This paper deals with the use of the Conjugate Gradient Method of function estimation for the simultaneous identification of two boundary conditions in natural convection inverse problems in cavities. The functions are estimated with no *a priori* information about its functional form. Irregular geometries in the physical domain are transformed into regular geometries in the computational domain, by using an elliptic scheme of numerical grid generation. Special emphasis is given in this paper to the validation of the numerical solution of the direct problem, which was obtained by finite-volumes. Such a numerical solution was validated by comparing the results obtained here with benchmark solutions available in the literature. An example of the inverse problem solution is given, involving the estimation of the boundary heat fluxes at two surfaces of an annular circular cavity.

1. Introduction

Forced and natural convection problems have been gaining the attention of groups aiming at the development of solution procedures for inverse problems, as well as of groups mainly involved with the physical/application aspects of this class of problems. Several recent works dealing with inverse convection problems can be found in references [1-28].

The use of simulated measurements has been widely used to verify the capabilities of an inverse problem solution procedure [1-31]. In such an approach, the direct problem is solved with *a priori* established values for the unknown parameters or functions. The solution of the direct problem then provides *exact* measurements to be used as input data for the inverse problem solution procedure. Note, however, that actual measurements generally contain errors. Therefore, in order to simulate actual measured data, random errors are added to the *exact* measurements. Standard statistical hypotheses generally assumed for the measurement errors include they being regarded as additive, uncorrelated, normally distributed, with zero mean and with constant and known standard-deviation [29-31]. By using simulated measurements obtained in such a manner, the inverse problem solution procedure shall be able to recover the values *a priori* established for the unknown parameters or functions. Different important issues can then be addressed with this approach, such as the stability of the solution procedure with respect to the measurement errors, as well as the design of the experiment, including the estimation of the number and position of sensors required for the inverse problem solution.

A fact usually overlooked when using simulated measurements for the inverse analysis is that errors in the mathematical model for the physical problem under picture, as well as in the solution technique for the direct problem, are neglected. Such is the case because the mathematical formulation and the solution technique for the direct problem, used to generate the simulated measurements, are the same used as part of the inverse problem solution procedure. In fact, for solving the inverse problem and for generating the simulated measurements, many analysts solve the direct problem disregarding the accuracy of its solution, which can result in unrealistic simulated measurements that may not be identified through the inverse analysis.

In this paper, we examine the simultaneous estimation of the boundary heat flux at two surfaces of a cavity, by using simulated temperature measurements taken in its interior. The fluid inside the cavity undergoes natural convection as a result of the prescribed boundary conditions. The natural convection problem is formulated in terms of generalized boundary-fitted coordinates [32], by using Boussinesq's approximation. The irregular geometry in the physical domain is transformed into a regular geometry in the computational domain, so that one single formulation can be used for the solution of inverse problems in cavities of different geometries. For the solution of the inverse problem, we consider the conjugate gradient method of function estimation with adjoint problem [30,31]. The direct problem, as well as the auxiliary problems required by this method, are numerically solved with finite-volumes, by utilizing the WUDS[33] interpolation scheme. The SIMPLEC method [34] was used for the treatment of the pressure-velocity coupling, for the computation of the velocity and pressure fields, on co-located grids. Special emphasis is given below for the validation of the numerical solution of the direct problem. The results obtained here are compared to benchmark solutions available in the literature for square cavities, as well as for annular circular and elliptical cavities. The solution for the inverse problem of simultaneously estimating two boundary heat fluxes at the surface of an annular circular cavity is presented.

2. Physical Problem and Mathematical Formulation

The physical problem under picture in this paper involves the transient laminar natural convection of a fluid inside a two-dimensional cavity. The surface for the cavity is assumed to be defined by four surfaces, which are transformed into the computational domain as the surfaces $x = 1$, $x = M$, $h = 1$ and $h = N$. The fluid is initially at rest and at the temperature T_c, which is also assumed to be the temperature at the surface $h = 1$. At time zero, the surface at $h = N$ has its temperature changed to T_h. The other two surfaces are kept insulated. The fluid properties are assumed constant, except for the density in the buoyancy term, where we consider Boussinesq's approximation valid. The mathematical formulation for this physical problem can be written in terms of the following conservation equation in the generalized Cartesian coordinates:

$$\frac{\partial(J r f)}{\partial t}+\frac{\partial\left(\tilde{U} r f\right)}{\partial x}+\frac{\partial\left(\tilde{V} r f\right)}{\partial h}=\frac{\partial}{\partial x}\left\{J\,\Gamma^{f}\left[a\frac{\partial f}{\partial x}+d\frac{\partial f}{\partial h}\right]\right\}+\frac{\partial}{\partial h}\left\{J\,\Gamma^{f}\left[d\frac{\partial f}{\partial x}+b\frac{\partial f}{\partial h}\right]\right\}+JS \tag{1}$$

where

$$\begin{gathered} a = x_x^2 + x_y^2; \quad b = h_x^2 + h_y^2; \quad d = x_x h_x + x_y h_y; \quad J = x_x y_h - x_h y_x; \\ \tilde{U} = J\left(u x_x + v x_y\right); \quad \tilde{V} = J\left(u h_x + v h_y\right) \end{gathered} \tag{2.a-f}$$

We note that $\tilde{U}$ and $\breve{V}$ denote the contravariant velocities in the x and h directions, respectively, while J defines the Jacobian of the transformation from the physical domain into the computational domain. The general conservation variable, as well as the diffusion coefficient and the source-term, can be found in table 1 for the mass, momentum and energy conservation equations. These equations are solved, subjected to the following boundary and initial conditions.

$$u = v = 0 \qquad \text{at } x = 1,\ x = M,\ h = 1 \text{ and } h = N,\ \text{for } t > 0 \tag{3.a}$$

$$D_{11}\frac{\partial u}{\partial x}+D_{12}\frac{\partial u}{\partial h}=0 \qquad \text{at } x = 1 \text{ and } x = M,\ \text{for } t > 0;\ \text{for cases with symmetry at } x = 1 \text{ and } x = M \tag{3.b}$$

$$D_{11}\frac{\partial v}{\partial x}+D_{12}\frac{\partial v}{\partial h}=0 \qquad \text{at } x = 1 \text{ and } x = M,\ \text{for } t > 0;\ \text{for cases with symmetry at } x = 1 \text{ and } x = M \tag{3.c}$$

$$T = T_h \qquad \text{at } h = N,\ \text{for } t > 0 \tag{3.d}$$

$$T = T_c \qquad \text{at } h = 1\ ,\ \text{for } t > 0 \tag{3.e}$$

$$D_{11}\frac{\partial T}{\partial x}+D_{12}\frac{\partial T}{\partial h}=0 \qquad \text{at } x = 1 \text{ and } x = M\ ,\ \text{for } t > 0 \tag{3.f}$$

$$u = v = 0 \qquad \text{for } t = 0 \text{ in the region} \tag{3.g}$$

$$T = T_c \qquad \text{for } t = 0 \text{ in the region} \tag{3.h}$$

where

$$D_{11} = a\, J\, \Gamma^f ; \quad D_{12} = b\, J\, \Gamma^f \tag{4.a,b}$$

Table 1: Conservation variable, diffusion coefficient and source-term.

Conservation of	f	G^f	S^f
Mass	1	0	0
x-momentum	u	m	$-\partial P/\partial x$
y-momentum	v	m	$-\partial P/\partial y - r\, g[1-b(T-T_{\text{ref}})]$
Energy	T	K/C_p	0

We note in table 1 that the positive y axis in the physical domain is supposed to be aligned with the opposite direction of gravitational acceleration vector.

3. Direct Problem and Inverse Problem

The *direct problem* associated with the mathematical formulation given by equations (1-4) involves the determination of the transient velocity and temperature fields in the cavity, from the knowledge of the cavity geometry, of the physical properties and of the initial and boundary conditions. Appropriately formulated direct problems are mathematically classified as *well-posed*. The solution of a well-posed problem must satisfy the conditions of existence, uniqueness and stability with respect to the input data [29-31].

Inverse heat transfer problems involve the estimation of at least one of the quantities required for the well-posedness of the direct problem, by using velocity or temperature measurements. The *inverse problem* of interest in this work involves the simultaneous estimation of the boundary heat fluxes $q_1(x,t)$ and $q_N(x,t)$, at the surfaces $h = 1$ and $h = N$, by using temperature measurements taken inside the cavity. Differently from direct problems, inverse problems are mathematically classified as *Ill-posed*. The existence of a solution for an inverse heat transfer problem may be assured by physical reasoning. On the other hand, the uniqueness of the solution of inverse problems can be mathematically proved only for some special cases. Also, the inverse problem is very sensitive to random errors in the measured input data, thus requiring special techniques for its solution in order to satisfy the stability condition [29-31]. In fact, a successful solution of an inverse problem generally involves its reformulation as an approximate well-posed problem and makes use of some kind of regularization (stabilization) technique. In several methods, the solution for the inverse problem is obtained through the minimization of an L_2 norm in the space where the unknown quantity belongs to. For the solution of the inverse problem under picture in this work we consider the minimization of the following functional:

$$F[q_1(x,t), q_N(x,t)] = \frac{1}{2}\int_{t=0}^{t_f} \sum_{s=1}^{S} [T(x_s, h_s, t; q_1, q_N) - m_s(t)]^2 \, dt \tag{5}$$

where t_f denotes the final time, S is the number of sensors used in the analysis, while $m_s(t)$ and $T(x_s, h_s, t; q_1, q_N)$ are the measured and estimated temperatures, respectively, at the measurement positions (x_s, h_s), for $s = 1,\ldots,S$. The estimated temperatures are obtained from the solution of the direct problem, by using estimates for the boundary heat fluxes $q_1(x,t)$ and $q_N(x,t)$.

We use simulated temperature measurements for the solution of the present inverse problem. These measurements are obtained from the solution of the direct problem for known boundary temperatures T_h and T_c , at the surfaces

$h = N$ and $h = 1$, respectively. The measurements obtained in such a manner are considered as *exact* ($m_{s,exact}(t)$) and, in order to simulated measurement errors, a random term is added to them in the form:

$$m_s(t) = m_{s,exact}(t) + w\,s \tag{6}$$

where w is a random variable with normal distribution, zero mean and unitary standard-deviation and s is the standard-deviation of the measurement errors.

Note that the simulated measurements are generated with the solution of the direct problem as given by equations (1-4), which involve first-kind boundary conditions for the surfaces $h = 1$ and $h = N$. On the other hand, since the inverse analysis is concerned with the simultaneous estimation of the heat fluxes at these two boundaries, the direct problem is formulated with second-kind boundary conditions at the surfaces $h = 1$ and $h = N$, for the solution of the inverse problem. Since the physical problem involves transient natural convection with the surfaces $h = 1$ and $h = N$ maintained at fixed temperatures, the heat fluxes at these two boundaries vary in time and along the boundary. The recovery of this type of function *via* inverse analysis is quite involved and it requires transient measurements taken at several locations inside the cavity.

4. Solution Techniques for the Direct and Inverse Problems

In this work, the direct problem was solved *via* finite-volumes, by utilizing the WUDS[33] interpolation scheme. The SIMPLEC method [34] was used for the treatment of the pressure-velocity coupling, for the computation of the velocity and pressure fields on co-located grids.

For the solution of the inverse problem through the minimization of the functional given by equation (5), we utilized the conjugate gradient method of function estimation with adjoint problem [30,31]. The basic steps of this method include [31]: (i) Direct Problem, (ii) Inverse Problem, (iii) Sensitivity Problems, (iv) Adjoint Problem, (v) Gradient Equations, (vi) Iterative Procedure, (vii) Stopping Criterion and (viii) Computational Algorithm. Like the direct problem, the auxiliary sensitivity and adjoint problems, as well as the gradient equations, were formulated in terms of boundary-fitted coordinates and an elliptic scheme of numerical grid generation was utilized in order to define the transformation from the physical domain into the computational domain[32]. Therefore, the present formulation is general and can be utilized for the solution of boundary inverse natural convection problems in cavities of any shape.

For the sake of brevity, we omit here details of the basic steps of the conjugate gradient method, but they can be readily found in references [27, 31], as applied to the solution of inverse heat conduction and forced convection problems in irregularly shaped two-dimensional regions. We note, however, that we used in this work Powell-Beale's version of the conjugate gradient method [35], which allows for restarting of the iterative procedure when the search direction is not sufficiently downhill and when the gradients at successive iterations are too far from orthogonality. We also tested the most common versions of the conjugate gradient method by Flecther-Reeves and by Polak-Ribiere [30,31,35], but, for several cases, they resulted in non-convergence of the iterative procedure because of the strong non-linear character of the physical problem. As in reference [35], we found Powell-Beale's version of the conjugate gradient method more stable and robust than these two other versions.

We note that the iterative procedure of the conjugate gradient method is not capable of providing by itself regularized solutions for inverse problems. In fact, it is generally observed that the random errors present on the measured variables are amplified for the solution of the inverse problem, as a result of its ill-posed character, when estimated temperatures approach the measured ones during the minimization of the functional (5). However, the use of the conjugate gradient method may result on stable solutions if the *Discrepancy Principle* [30,31] is used to specify the tolerance for the stopping criterion of the iterative procedure. In the Discrepancy Principle, the solution is assumed to be sufficiently accurate when the difference between measured and estimated temperatures is of the order of magnitude of the measurement errors.

5. Results and Discussions

In this work, we focus on the validation of the numerical solution for the direct problem, which is used to generate the simulated measurements, as well as part of the solution procedure for the inverse problem. With this objective in mind, we compare below our numerical results with *benchmark* solutions available in the literature for natural convection problems inside cavities of different shapes, including: (i) Square cavity; (ii) Annular elliptical cavity; and (iii) Annular circular cavity.

5.1 Square Cavity

For this case, we consider a square cavity as illustrated by figure 1. Air is supposed to be the fluid inside the cavity with properties: r = 1.19 kg/m^3, m = 1.8x10^{-5} kg/ms, b = 0.00341 K^{-1}, Pr = 0.71, K = 0.02624 W/mK, C_p = 1035.02 J/kg ^{o}C. The vertical walls are maintained at temperatures T_h = 12 ^{o}C and T_c = 2 ^{o}C. We present below the results obtained for a Rayleigh number of 10^6, so that the dimension of the cavity is given by H = L = 0.099 m. As benchmark results for this problem we use the recent steady-state solution obtained by Leal et al [36] by using the Generalized Integral Transform Technique.

Table 2 presents a comparison of the results obtained here with those of reference [36] for the maximum x and y velocities and for the average Nusselt number at the hot boundary. Table 2 shows the results obtained with meshes with 30x30, 50x50 and 80x80 equally spaced volumes. The time step used was 5x10^{-5} s, which was the largest one tested that resulted on stable solutions. The transient problem was solved until t = 100 s, when the steady-state was reached. As expected, the discrepancy between our solution and that of reference [36] decreases, as the number of volumes used for the discretization increases. In fact, quite accurate results were obtained with 80x80 volumes in the mesh, even for such a high value for the Rayleigh number.

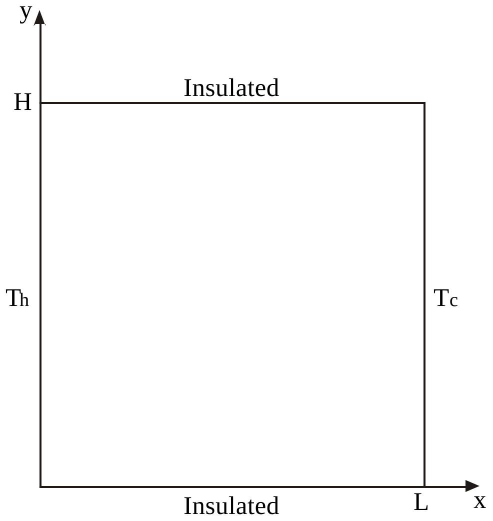


Figure 1: Square cavity

Table 2: Results obtained for the square cavity

	Ref. [36]	Grid 30x30	Error (%)	Grid 50x50	Error (%)	Grid 80x80	Error (%)
$u_{\text{max.}}$	64.830	61.995	4.4	63.957	1.3	64.244	0.9
$v_{\text{max.}}$	220.600	204.025	7.5	212.197	3.8	214.615	2.7
$Nu_{\text{avg.}}$	8.825	8.747	0.9	8.789	0.4	8.798	0.3

5.2 Annular Elliptical Cavity

The second case examined for the validation of the numerical solution of the direct problem was natural convection problem inside an annular elliptical cavity, as depicted in figure 2. As the benchmark results, we consider those obtained by Elshamy et al [37], by using finite-volumes and a stream function-vorticity formulation. Air was considered as the fluid inside the cavity, with the same values for the physical properties considered above for the square cavity. Some geometrical parameters defining the cavity for the case examined (see figure 2) are: A_0 = 0.0230 m, A_i = 0.0108 m, B_0 = 0.0214 m and B_i = 0.0055 m. The temperatures at the walls were taken as T_h = 34 °C and T_c = 10 °C. The Rayleigh number for this case is 10^4, where the characteristic length was taken as $B_0 - B_i$.

Figure 3 presents a comparison of the results obtained here with those of Elshamy et al [37] for the steady-state local Nusselt number at the inner and outer surface of the cavity. For the numerical results computed here, we have used 67x67 volumes and a time increment of 5×10^{-3} s. The computations were run until steady-state was reached at 100 s. An examination of figure 3 reveals a very good agreement between our results and those of Elshamy et al [37]. Indeed, the discrepancies between our results and those of reference [37] were smaller than 0.3% for the average Nusselt number.

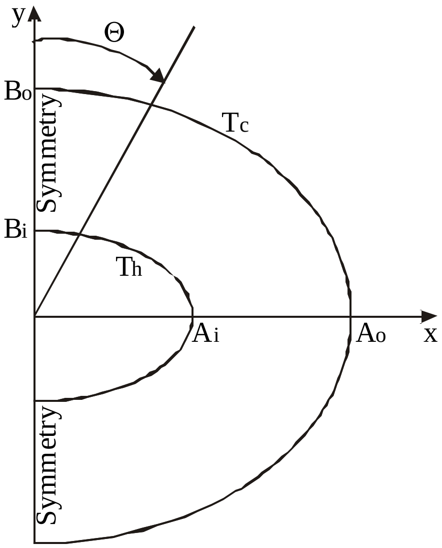


Figure 2: Annular Elliptical Cavity

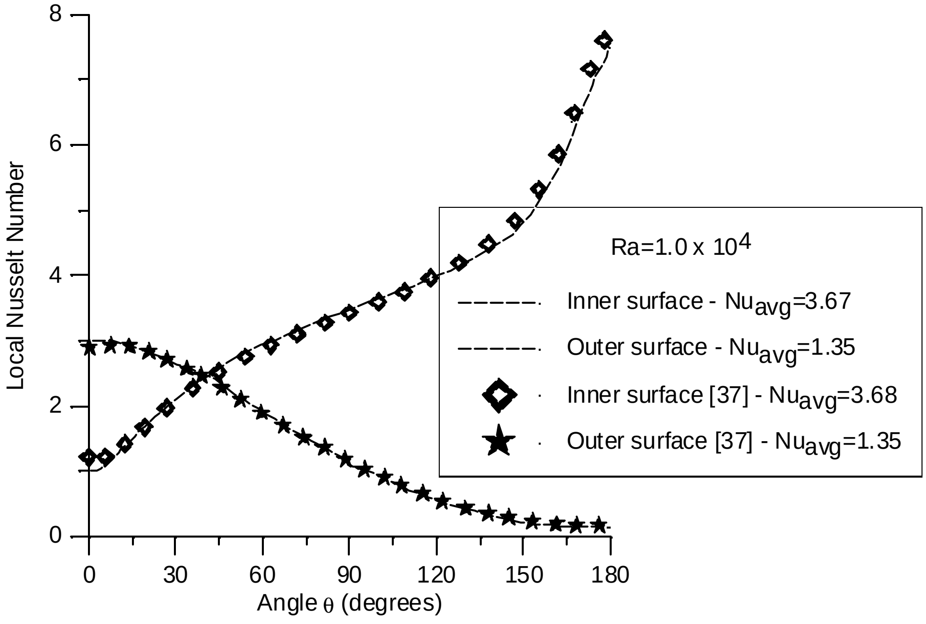


Figure 3: Local Nusselt number for the Annular Elliptical Cavity

5.3 Annular Circular Cavity

We now turn our attention to an annular circular cavity, as depicted in figure 4. The results obtained here for natural convection of air (with the same physical properties used above for the square cavity) were compared to those of Pereira et al [38] obtained with the Generalized Integral Transform Technique. The geometrical dimensions were taken as R_1= 0.0228 m and R_2 = 0.0594 m, while the temperatures at the walls were taken as T_h = 30 °C and T_c = 20 °C. For this case, the Rayleigh number is 5×10^4, where the characteristic length used was R_2 - R_1.

Figure 5 presents a comparison of the results obtained here with those of reference [38] for the steady-state local Nusselt number at the hot and cold surfaces. The agreement between the two solutions is excellent. The numerical results were obtained by using a mesh with 80x80 volumes and an increment in time of 5×10^{-3} s. The computations were run until steady-state was reached at 100 s.

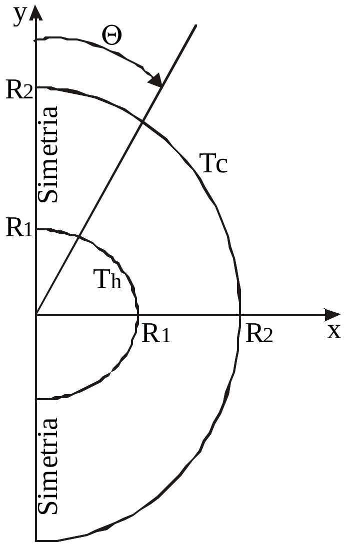


Figure 4: Annular Circular Cavity

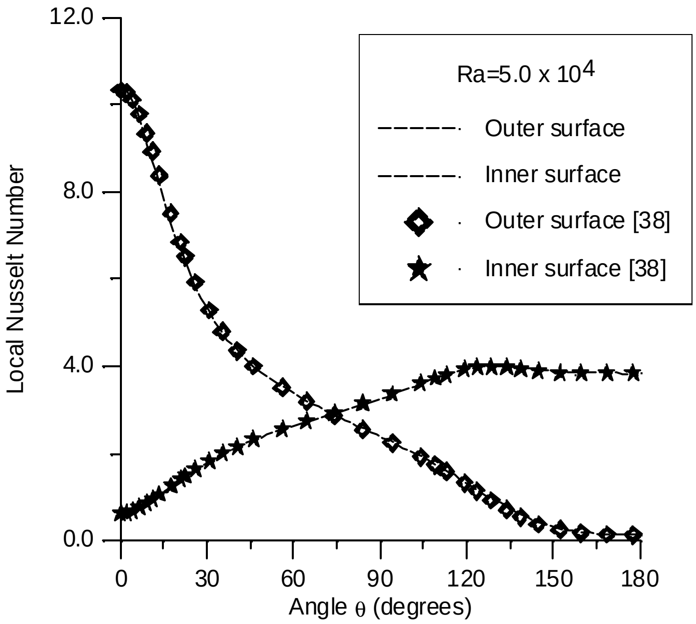
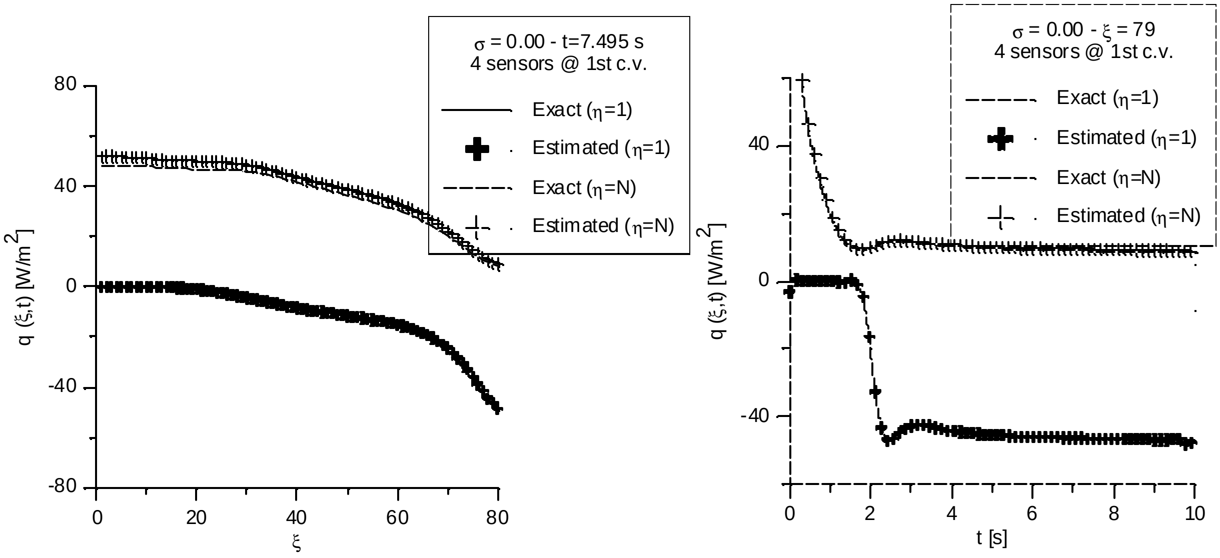


Figure5: Local Nusselt Number for Annular Circular Cavity

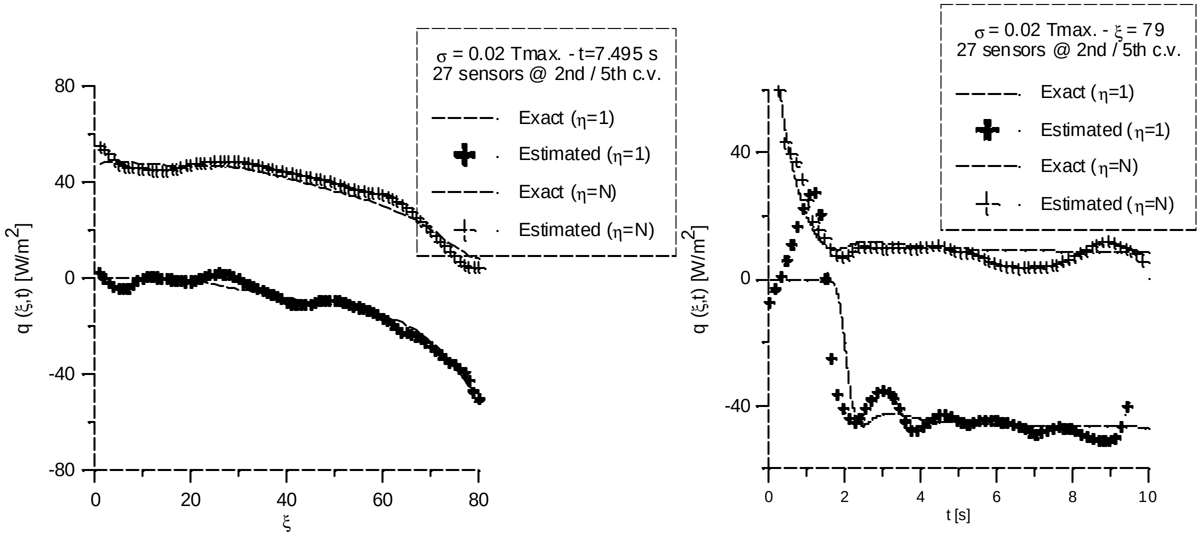
After validating the numerical solution for the direct problem, we illustrate the inverse analysis of simultaneously estimating the boundary heat fluxes at the two walls of the annular circular cavity. We consider an idealistic test-case involving the transient measurements of only 4 sensors located at the control-volumes next to the surfaces with unknown heat flux. Also, the simulated transient measurements were assumed as errorless. The results obtained for the estimated heat fluxes at the two surfaces in this case are illustrated in figure 6, which shows the spacewise variation of the heat fluxes at time $t = 7.5$ s, and the timewise variation of the heat fluxes at $\xi=79$. The initial guess used for the conjugate gradient method was a quite small (0.1 W/m^2) heat flux at the two boundaries. We note in these figures that the unknown heat fluxes were correctly recovered, despite the fact that the initial guess is quite far from the exact solution



(a) (b)

Figure 6: Inverse Problem Solution without measurements errors

Finally, in figure 7, it is shown a case involving the estimative with measurements errors. In this case, we utilized 27 sensors, located at the 2nd and 5th control volumes next to the boundaries $\eta=1$ and $\eta=N$, respectively. The standard deviation σ was chosen equal to 2 % of the maximum temperature (in this case, $T_h=30$ °C). We note that, even with errors in the measurements, the estimative still is good.



(a) (b)

Figure 7: Inverse Problem Solution with measurements errors

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